

Exponential Functions Study Guide

Name _____

1. Answer each question below about the following sequence: 17.5, 8.75, 4.375, 2.1875,

- a. Write the recursive formula for the sequence. a. _____
- b. Write the explicit formula for the sequence. b. _____
- c. Find the 50th term of the sequence. Write your answer in scientific notation. Round to the nearest tenth. c. _____

2. The function $a(n) = 3 \cdot 2^n$ represents the value of the n th term in a sequence. What is the **sum** of the first and fifth terms of the sequence?

2. _____

3. A scientist measures a cluster of bacteria made up of 20 cells. After 6 hours, the cluster had doubled to 40 cells. After 12 hours, it had doubled to 80 cells. How many cells of bacteria will there be after 3 days?

3. _____

4. Let a sequence be defined recursively as: $a_1 = 5$; $a_n = 3 \cdot a_{n-1}$

- a. Generate the first 5 terms of the sequence. a. _____
- b. Write the explicit formula for the sequence given. b. _____
- c. Find the sum of the 10th and 13th terms. c. _____
- d. Write the exponential function for the sequence based on the zero term. d. _____

5. Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 5 every **hour**. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of **days**. Make sure to use function notation.

5. _____

6. For each exponential function circle whether it is growth or decay, identify the initial value, circle whether it is a decrease or increase and identify and by what **percentage** does the value decrease/increase every year.

- a. $f(x) = 3.34(1.67)^x$ Growth/Decay, Initial Value: _____, Increase/Decrease, Percentage: _____
- b. $f(x) = 8(0.54)^x$ Growth/Decay, Initial Value: _____, Increase/Decrease, Percentage: _____
- c. $f(x) = 4.5(4)^x$ Growth/Decay, Initial Value: _____, Increase/Decrease, Percentage: _____

7. The population of rabbits in a national forest has been declining by $1/20$ each year since 2003 when its population was measured at 4,578 rabbits.

- a. Write an equation for the population t years after 2003. a. _____
- b. Use the equation to predict the population of the rabbits in the forest in 2015. Round to the nearest whole number. b. _____

8. The table to the right shows the ending balance of a college savings account for each year listed.

Year	Amount
2008	\$10,991
2009	\$11,343
2010	\$11,706
2011	\$12,080

- a. If the savings account was opened in 2005, what was the initial amount invested in the college savings account? Round to the nearest whole number. a. _____
- b. Use your initial value from part a to help you write an exponential equation that models the amount in the savings account n years since 2005. Round "b" to the nearest thousandth. b. _____
- c. If the student that the account was created for is to enter college in 2017, how much money will be in the account? Round to the nearest hundredth. c. _____

9. The current population of the world is about 7,755,302,185. It is estimated that the population of the world is increasing and has been increasing at an average annual rate of 1.3%. At this rate, what was the estimated population for the world 14 years ago?

9. _____

10. Wilma and Walder's Weaving Wanders bought a piece of weaving equipment for \$60,000. It is expected to depreciate at an average rate of 10% per year. Between which 2 consecutive years will the equipment be worth half of its original value, if they bought the equipment in 2019?

10. _____

11. Three functions are shown below.

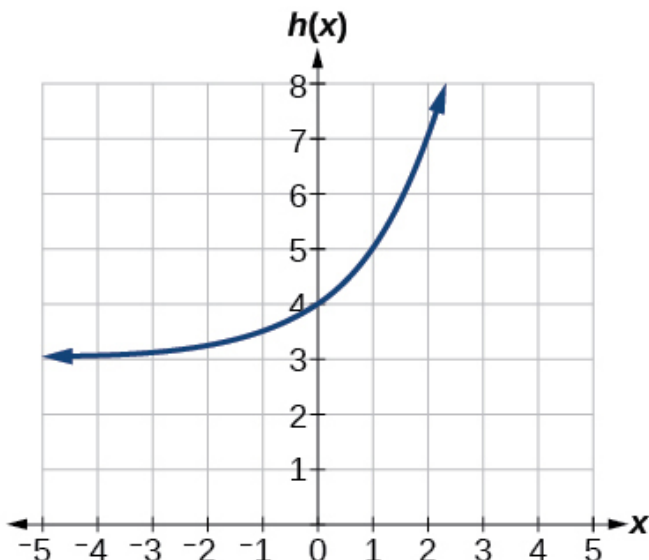
$$g(x) = 7x + 2$$

$$h(x) = 2x^2 + 3x + 1$$

$$f(x) = \frac{1}{4} \cdot 2^x$$

- a. What is the largest integer value of x such that $h(x) \leq g(x)$? a. _____
- b. What is the largest integer value of x such that $f(x) \leq g(x)$? b. _____
- c. What is the largest integer value of x such that $f(x) \leq h(x)$? c. _____

12. For $h(x) = 2^x + 3$ graphed below answer all of the following:



- a. What is the domain? _____
- b. What is the range? _____
- c. How many x-intercepts? _____
- d. What is the y-intercept? _____
- e. What is the asymptote? _____
- f. What's the average rate of change for $h(x) = 2^x + 3$ from $1 \leq x \leq 3$? _____
- g. Describe the end behavior by filling in each blank:
- As x increases, $h(x)$ approaches _____.
 - As x decreases, $h(x)$ approaches _____.
- h. Evaluate $h(-4)$ _____
- i. When $h(x) = 5$, what is x ? _____

13. Identify each situation that models an exponential function by writing the letter(s) in the blank to the right.
- Your parents give you a new chore each week. In return, your allowance increases by \$5 each week.
 - You put your allowance money in a CD that appreciates 1.25% each year.
 - Each work out, your trainer works you harder by increasing your laps by 1.
 - There is a zombie apocalypse and each day the zombies split in thirds to create new zombies.
 - The value of an old boat declines by \$20 each day.
 - The value of real estate increases by 17% each month.
 - The half-life of uranium.
 - The distance a car travels as a function of the time.
13. _____

14. Give the equation for the function given in the table by running an exponential regression.

x	3	5	10	13
f(x)	54	486	118098	3188646

Equation: _____

x	-5	-1	3	5
g(x)	$\frac{512}{243}$	$\frac{2}{3}$	$\frac{27}{128}$	$\frac{243}{2048}$

Equation: _____

15. Every ten years, the Census counts how many people are living in every town in the United States.
- The 2010 Census showed that 1,000 people were living in Appleville, and 4,000 people were living in Bridgetown.
 - The population of Appleville is predicted to double every ten years.
 - The population of Bridgetown is predicted to increase by 1,000 every ten years.

If the predictions come true, what will be the first census year that will show Appleville with a larger population than Bridgetown?

15. _____

16. You are offered a 30-day trial period at your dream job. However, the owner of the company is a little unusual and retention focused; so, the pay options are non-traditional. They offer you 3 different pay options for the 30 days:
- Option 1:** \$60,000 a day
 - Option 2:** If you work a total of one day, you make one penny. If you work a total of two days, you make two pennies. If you work a total of three days, you make four pennies. If you work a total of four days, you make eight pennies, etc.
 - Option 3:** If you work a total of one day, you get \$1. If you work a total of two days, you make \$4. If you work a total of three days, you get \$9. If you work a total of four days, you get \$16, etc. It continues this way for 30 days and then once you've completed the 30 days you receive a completion bonus of \$500,000.
- Decide which payment option you would like to take based on earning the most money within the 30 days.

a. _____
 - If you were only working 20 days, which option is the most lucrative?

b. _____
 - On what day does your choice in part a become the best option over the other two options?

c. _____

Exponential Functions Study Guide

Name Key

1. Answer each question below about the following sequence: 17.5, 8.75, 4.375, 2.1875, ...

a. Write the recursive formula for the sequence.

a. $a_1 = 17.5; a_n = \frac{1}{2} \cdot a_{n-1}$

b. Write the explicit formula for the sequence.

b. $a_n = 17.5 \left(\frac{1}{2}\right)^{n-1}$

c. Find the 50th term of the sequence. Write your answer in scientific notation. Round to the nearest tenth.

$a_{50} = 17.5 \left(\frac{1}{2}\right)^{50-1}$

c. 3.1×10^{-14}

2. The function $a(n) = 3 \cdot 2^n$ represents the value of the n th term in a sequence. What is the sum of the first and fifth terms of the sequence?

$a(1) = 3 \cdot 2^1 = 6$
 $a(5) = 3 \cdot 2^5 = 96$
 $6 + 96 = 102$

2. 102

3. A scientist measures a cluster of bacteria made up of 20 cells. After 6 hours, the cluster had doubled to 40 cells. After 12 hours, it had doubled to 80 cells. How many cells of bacteria will there be after 3 days?

n	1	2	3	4	5
a_n	40	80	160	320	640

6 hrs 12 hrs 18 hrs
 $a_0 = 20$
 $a_1 = 40; a_n = 40 \cdot 2^{n-1}$
 3 days has 12 groups of 6 in it. $= 40 \cdot 2^{12-1} = 81920$

3. 81,920 cells

4. Let a sequence be defined recursively as: $a_1 = 5; a_n = 3 \cdot a_{n-1}$

a. Generate the first 5 terms of the sequence.

a. 5, 15, 45, 135, 405

b. Write the explicit formula for the sequence given.

b. $a_n = 5 \cdot 3^{n-1}$

c. Find the sum of the 10th and 13th terms.

c. 2755620

d. Write the exponential function for the sequence based on the zero term.

n	0	1	2	3
a_n	$\frac{5}{3}$	5	15	45

$a_0 = \frac{5}{3}$
 $\div 3 \leftarrow \times 3$

d. $f(n) = \frac{5}{3} \cdot 3^n$

5. Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 5 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days. Make sure to use function notation.

n	1	2	3
a_n	5	25	125

$a_n = 5 \cdot 5^{n-1}$

n	24n
hours	days

 $\times 24$
 $a_n = 5 \cdot 5^{24n-1}$
 $n = \text{days}$

5. $f(n) = 5 \cdot 5^{24n-1}$

6. For each exponential function circle whether it is growth or decay, identify the initial value, circle whether it is a decrease or increase and identify and by what percentage does the value decrease/increase every year.

a. $f(x) = 3.34(1.67)^x$ Growth/Decay, Initial Value: 3.34, Increase/Decrease, Percentage: 67%

b. $f(x) = 8(0.54)^x$ Growth/Decay, Initial Value: 8, Increase/Decrease, Percentage: 53%

c. $f(x) = 4.5(4)^x$ Growth/Decay, Initial Value: 4.5, Increase/Decrease, Percentage: 300%

7. The population of rabbits in a national forest has been declining by $\frac{1}{20}$ each year since 2003 when its population was measured at 4,578 rabbits.

Sustaining $\frac{19}{20}$

a. Write an equation for the population t years after 2003.

a. $y = 4578 \left(\frac{19}{20}\right)^t$

b. Use the equation to predict the population of the rabbits in the forest in 2015.

$x=0$ 2003 \rightarrow 2015
 $+12$ +12
 $x=12$ $y = 4578 \left(\frac{19}{20}\right)^{12}$

b. 2474

13. Identify each situation that models an exponential function by writing the letter(s) in the blank to the right.

- a. Your parents give you a new chore each week. In return, your allowance increases by \$5 each week.
- b.** You put your allowance money in a CD that appreciates 1.25% each year.
- c. Each work out, your trainer works you harder by increasing your laps by 1.
- d.** There is a zombie apocalypse and each day the zombies split in thirds to create new zombies.
- e. The value of an old boat declines by \$20 each day.
- f.** The value of real estate increases by 17% each month.
- g.** The half-life of uranium.
- h. The distance a car travels as a function of the time.

13. b, d, f, g

14. Give the equation for the function given in the table by running an exponential regression.

x	3	5	10	13
f(x)	54	486	118098	3188646

Use calc!

Equation: $f(x) = 2 \cdot 3^x$

x	-5	-1	3	5
g(x)	$\frac{512}{243}$	$\frac{2}{3}$	$\frac{27}{128}$	$\frac{243}{2048}$

Equation: $g(x) = \frac{1}{2} \left(\frac{3}{4}\right)^x$

15. Every ten years, the Census counts how many people are living in every town in the United States.

- The 2010 Census showed that 1,000 people were living in Appleville, and 4,000 people were living in Bridgetown.
- The population of Appleville is predicted to double every ten years. **x2 exponential**
- The population of Bridgetown is predicted to increase by 1,000 every ten years. **+1000 linear**

If the predictions come true, what will be the first census year that will show Appleville with a larger population than Bridgetown?

let $x =$ every 10 yrs
 $A(x) = 1000(2)^x$
 $B(x) = 4000 + 1000x$
 Used calc/desmos to find intersection of (2.76, 6756.2)
 Not beginning, round to 3, so +30 yrs. 2010+30 = 2040
 15. 2040

16. You are offered a 30-day trial period at your dream job. However, the owner of the company is a little unusual and retention focused; so, the pay options are non-traditional. They offer you 3 different pay options for the 30 days:

let $x =$ # of days you work

- **Option 1:** \$60,000 a day $y = 60000x$
- **Option 2:** If you work a total of one day, you make one penny. If you work a total of two days, you make two pennies. If you work a total of three days, you make four pennies. If you work a total of four days, you make eight pennies, etc. $y = .01 \cdot 2^{x-1}$
- **Option 3:** If you work a total of one day, you get \$1. If you work a total of two days, you make \$4. If you work a total of three days, you get \$9. If you work a total of four days, you get \$16, etc. It continues this way for 30 days and then once you've completed the 30 days you receive a completion bonus of \$500,000.

$y = x^2$ OR at 30 days $y = x^2 + 500000$

a. Decide which payment option you would like to take based on earning the most money within the 30 days.

let $x = 30$

a. Option 2

b. If you were only working 20 days, which option is the most lucrative?

let $x = 20$

b. Option 1

c. On what day does your choice in part a become the best option over the other two options?

Intersection (28.34, 1700482.12)

c. Day 29

Run Regression

Year	Amount
n	A
2008	\$10,991
+3	
2009	\$11,343
+4	
2010	\$11,706
+5	
2011	\$12,080
+6	

8. The table to the right shows the ending balance of a college savings account for each year listed.

a. If the savings account was opened in 2005, what was the initial amount invested in the college savings account? Round to the nearest whole number. a. \$ 10,000

b. Use your initial value from part a to help you write an exponential equation that models the amount in the savings account n years since 2005. Round "b" to the nearest hundredth.

b. $A = 10000(1.032)^n$

c. If the student that the account was created for is to enter college in 2017, how much money will be in the account? Round to the nearest hundredth.

c. \$ 14593.40
 $2017 - 2005 = 12$ $10000(1.032)^{12}$

9. The current population of the world is about 7,755,302,185. It is estimated that the population of the world is increasing and has been increasing at an average annual rate of 1.3%. At this rate, what was the estimated population for the world 14 years ago?

$7755302185(1.013)^{-14}$

9. 6,472,416,997
 people

10. Wilma and Walder's Weaving Wanders bought a piece of weaving equipment for \$60,000. It is expected to depreciate at an average rate of 10% per year. Between which 2 consecutive years will the equipment be worth half of its original value?

$30000 = 60000(0.9)^x$

Between

used calc/desmos to find intersection of
 $(6.57, 30000)$

10. 2025 and 2026

11. Three functions are shown below.

$g(x) = 7x + 2$

$h(x) = 2x^2 + 3x + 1$

$f(x) = \frac{1}{4} \cdot 2^x$

or graph in calc & use table!

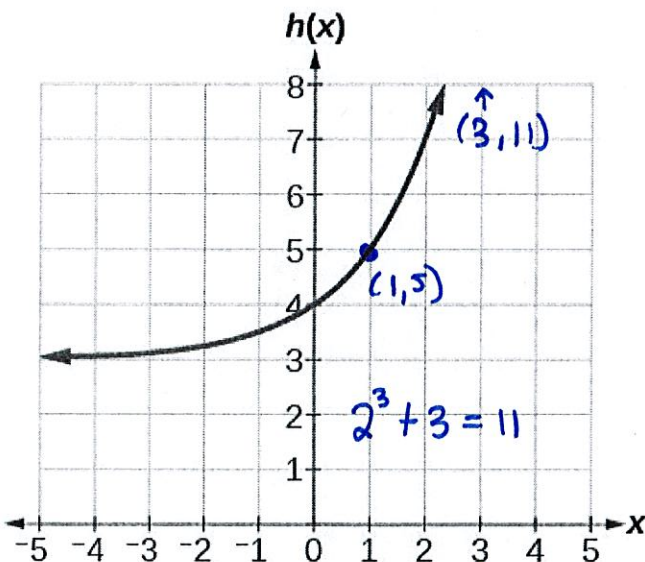
Graph in calc or desmos to find intersections.

a. What is the largest integer value of x such that $h(x) \leq g(x)$? (2.2, 17.6) a. 2

b. What is the largest integer value of x such that $f(x) \leq g(x)$? (7.8, 56.8) b. 7

c. What is the largest integer value of x such that $f(x) \leq h(x)$? (9.7, 222.3) c. 9

12. For $h(x) = 2^x + 3$ graphed below answer all of the following:



a. What is the domain? \mathbb{R}

b. What is the range? $y > 3$ $(3, \infty)$

c. How many x-intercepts? none

d. What is the y-intercept? $(0, 4)$

e. What is the asymptote? $y = 3$

f. What's the average rate of change for $h(x) = 2^x + 3$ from $1 \leq x \leq 3$? $\frac{\Delta y}{\Delta x} = \frac{6}{2}$ 3

g. Describe the end behavior by filling in each blank:

• As x increases, $h(x)$ approaches ∞

• As x decreases, $h(x)$ approaches 3

h. Evaluate $h(-4)$ $49/16$

i. When $h(x) = 5$, what is x? $x = 1$
 look at graph

h. $h(-4) = 2^{-4} + 3 = \frac{1}{2^4} + 3$