Quadratics Review

Name:_____

Part I – Calculator Inactive

1. Answer the following questions about the function $f(x) = 3x^2 - 15x - 18$.

- **a.** What is the axis of symmetry?
- **b.** What is the vertex?
- **c.** Is it a maximum or a minimum?
- **d.** Concave up/down?
- **e.** What's the value of the discriminant?
- **f.** Based on the discriminant, how many roots does the function have?
- **g.** What are the roots?
- **h.** What is the domain?
- **i.** What is the range?

2. Answer the following questions about the function $g(x) = -4x^2 - 8x + 3$.

- **a.** What is the axis of symmetry?
- **b.** What is the vertex?
- **c.** Is it a maximum or a minimum?
- **d.** Concave up/down?
- **e.** What's the value of the discriminant?
- **f.** Based on the discriminant, how many roots does the function have?
- **g.** What are the roots?
- **h.** What is the domain?
- **i.** What is the range?

3. Answer the following questions about the function $h(x) = 2x^2 - 6x - 20$.

- **a.** What is the axis of symmetry?
- **b.** What is the vertex?
- **c.** Is it a maximum or a minimum?
- **d.** Concave up/down?
- **e.** What's the value of the discriminant?
- **f.** Based on the discriminant, how many roots does the function have?
- **g.** What are the roots?
- **h.** What is the domain?
- **i.** What is the range?

4. Answer the following questions about the function $h(x) = 2x^2 - 6x - 20$?

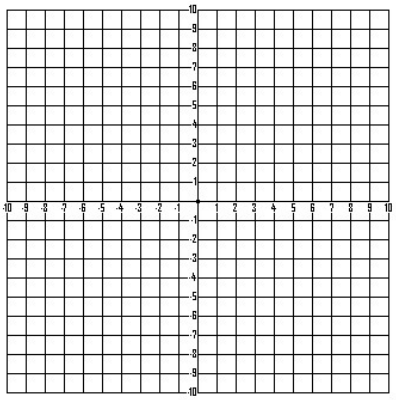
- **a.** Evaluate $h(3)$.
- **b.** Evaluate $h(-2)$
- **c.** If $h(x) = 28$, what is x?
- **d.** If $h(x) = 16$, what is x?
- **e.** What is the rate of change of $h(x)$ where $-2 \leq x \leq 0$?
- **f.** What is the rate of change of $h(x)$ where $3 \leq x \leq 6$?

5. For the functions in questions 1-3, which graph is the narrowest? How do you know?

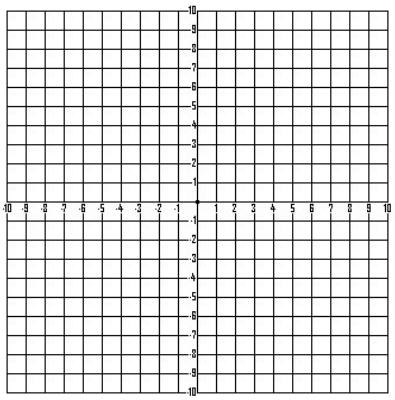
6. For the functions in questions 1-3, which graph is the widest? How do you know?

7. Graph f(x), g(x), and h(x) from problems 1-3 on the graphs below.

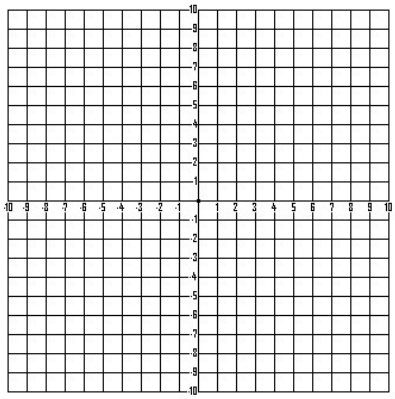
x	f(x)



x	g(x)



x	h(x)



8. An object is launched upwards at a speed of 128 ft/sec. It was launched from a height of 144 ft. How many seconds does it take the object to hit the ground?

_____

9. An object is launched upwards at a speed of 80 ft/sec. It was launched from a height of 32 ft. How many seconds does it take the object to be 128 ft off the ground?

_____

Part II - Calculator Active

8. If the roots of a quadratic are x = -5 and x = 2, work backward to create an original quadratic equation, in $y = ax^2 + bx + c$ form.

_____

9. The larger leg of a right triangle is 3 cm longer than its smaller leg. The hypotenuse is 6 cm longer than the smaller leg. What is the area of the triangle?

_____

10. A rectangular pool measures 5 yds by 6 yds. A concrete walkway of uniform width is constructed around the pool. The walkway and the pool together cover an area of 72 yd^2. Set up and solve an equation to show how wide the walkway is.

_____

11. When a driver needs to stop a car, the approximate stopping distance d (in feet) is given by the formula: $d = 0.05v^2 + 2.2v$, where v is the speed of the car (in miles per hour). Suppose a car travels 200 feet before stopping – how fast was the car traveling (to the nearest mph)?

_____

12. Find three consecutive ODD integers such that the product of the first and third integers minus the middle integer is 338.

_____

13. A basketball player shoots the ball with an upward velocity of 20 ft/s. The ball is 6 ft. above the floor when it leaves his hand. How long will it take for the ball to reach the rim of the basket, 10 ft. above the floor, on its way down?

_____

14. Suppose that the equation $V = 25.4x^2 – 512.6x + 4{,}500$ represents the value of a car from 1964 to 2002. What year did the car have the least value? (x = 0 in 1964)

_____

15. Which of the following represents the roots to the quadratic equation $b^2 – 16b + 64 = 19$, rounded to the nearest tenth?

_____

16. Solve the following system of equations: $\begin{cases} y = 2x + 1 \\ y = 2x^2 - 3 \end{cases}$

_____

17. Each of the "golden arches" at a McDonald's restaurant is in the shape of a parabola. Each arch is modeled by $h(x) = -x^2 + 6x$, where h(x) is the height of the arch (in feet) at a distance x (in feet) from one side.

A. Find the equation of the axis of symmetry. _____

B. Find the x-intercepts. How long is one arch across its base? _____

C. How high is the arch at its highest point? _____

18. Pyro-Tech shot a rocket off going upwards at 60 ft/sec. They shot the rocket from 80 feet above the ground.

A. How long will it take the rocket to reach its highest point? _____

B. How high will the rocket go? _____

C. How long after the launch will the rocket return to the ground? _____

Bonus: Starting with the standard form of quadratic equation, $ax^2+bx+c=0$, use completing the square to derive the proof of the quadratic formula.

Quadratics Review Name: Key

Part I – Calculator Inactive

1. Answer the following questions about the function $f(x) = 3x^2 - 15x - 18$.

a. What is the axis of symmetry? $x = 5/2$; $x = 2.5$
b. What is the vertex? $(5/2, -147/4)$ $(2.5, -36.75)$
c. Is it a maximum or a minimum? minimum
d. Concave up/down? concave up
e. What's the value of the discriminant? 441
f. Based on the discriminant, how many roots does the function have? $441 > 0$; So, 2 roots
g. What are the roots? $(-1, 0), (6, 0)$
h. What is the domain? $\mathbb{R}$
i. What is the range? $y \geq -147/4$; $y \geq -36.75$; $[-147/4, \infty)$

2. Answer the following questions about the function $g(x) = -4x^2 - 8x + 3$.

a. What is the axis of symmetry? $x = -1$
b. What is the vertex? $(-1, 7)$
c. Is it a maximum or a minimum? maximum
d. Concave up/down? concave down
e. What's the value of the discriminant? 112
f. Based on the discriminant, how many roots does the function have? $112 > 0$; So, 2 roots
g. What are the roots? $\left(\frac{-2+\sqrt{7}}{2}, 0\right), \left(\frac{-2-\sqrt{7}}{2}, 0\right)$
h. What is the domain? $\mathbb{R}$
i. What is the range? $y \leq 7$; $(-\infty, 7]$

3. Answer the following questions about the function $h(x) = 2x^2 - 6x - 20$.

a. What is the axis of symmetry? $x = 3/2$; $x = 1.5$
b. What is the vertex? $(3/2, -49/2)$; $(1.5, -24.5)$
c. Is it a maximum or a minimum? minimum
d. Concave up/down? concave up
e. What's the value of the discriminant? 196
f. Based on the discriminant, how many roots does the function have? 2 zeros
g. What are the roots? $(-2, 0), (5, 0)$
h. What is the domain? $\mathbb{R}$
i. What is the range? $y \geq -24.5$; $y \geq -49/2$ $[-49/2, \infty)$

4. Answer the following questions about the function $h(x) = 2x^2 - 6x - 20$?

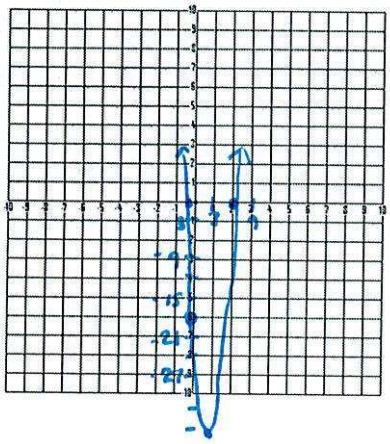
a. Evaluate h(3). $h(3) = -20$
b. Evaluate h(-2) $h(-2) = 0$
c. If h(x) = 28, what is x? $x = \frac{3 \pm \sqrt{105}}{2}$
d. If h(x) = 16, what is x? $x = -3$ or $x = 6$
e. What is the rate of change of h(x) where $-2 \leq x \leq 0$? -10
f. What is the rate of change of h(x) where $3 \leq x \leq 6$? 12

5. For the functions in questions 1-3, which graph is the narrowest? How do you know?
$g(x) = -4x^2 - 8x + 3$ is the narrowest b/c $|-4|$ is the largest a coefficient causing a faster rate of change & more vertical stretch.

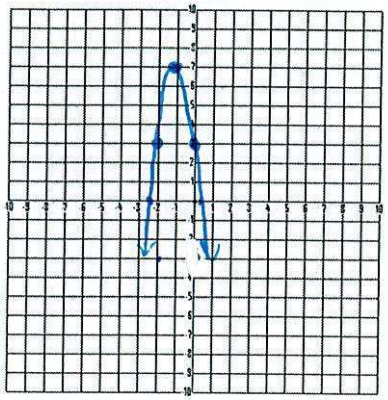
6. For the functions in questions 1-3, which graph is the widest? How do you know?
$h(x) = 2x^2 - 6x - 20$ is the widest because 2 is smaller than the absolute value of the other a coefficients.

7. Graph f(x), g(x), and h(x) from problems 1-3 on the graphs below.

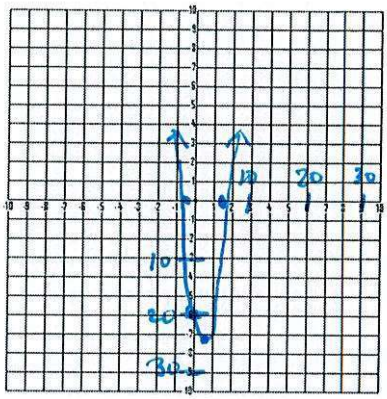
x	f(x)
-1	0
6	0
0	-18
2.5	-36.75



x	g(x)
-2.323	0
.323	0
-1	7
0	3



x	h(x)
-2	0
5	0
1.5	-24.5
0	-20



8. An object is launched upwards at a speed of 128 ft/sec. It was launched from a height of 144 ft. How many seconds does it take the object to hit the ground?

V_0 ↑ ↖ -16 h_0 ↗

↑ $h=0$

$h(x) = -16x^2 + 128x + 144$

$0 = -16x^2 + 128x + 144$

$x \neq -1$ OR $x = 9$

9 seconds

9. An object is launched upwards at a speed of 80 ft/sec. It was launched from a height of 32 ft. How many seconds does it take the object to be 128 ft off the ground?

V_0 h_0

↓ ↖ -16

$h=128$

$h(x) = -16x^2 + 80x + 32$

$128 = -16x^2 + 80x + 32$

$x = 2$ or $x = 3$

@

2 seconds

the object is @ 128 ft & again @ 3 seconds

Part II - Calculator Active

8. If the roots of a quadratic are x = -5 and x = 2, work backward to create an original quadratic equation, in $y = ax^2 + bx + c$ form.

$x = -5$ $\quad x = 2$

$+5 \quad +5 \quad -2 \quad -2$

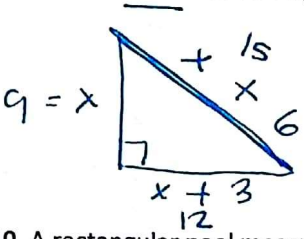
root $(x+5) = 0$ root $(x-2) = 0$

$(x+5)(x-2) = 0$

$x^2 - 2x + 5x - 10 = 0$

$x^2 + 3x - 10 = 0$ $\quad y = x^2 + 3x - 10$

9. The larger leg of a right triangle is 3 cm longer than its smaller leg. The hypotenuse is 6 cm longer than the smaller leg. What is the area of the triangle?



$a^2 + b^2 = c^2$

$x^2 + (x+3)^2 = (x+6)^2$

$x^2 + x^2 + 6x + 9 = x^2 + 12x + 36$

$x^2 - 6x - 27 = 0$

$(x+3)(x-9) = 0$

$x = -3$ OR $x = 9$

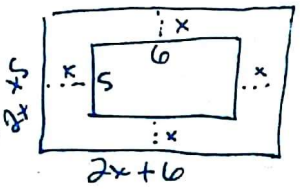
$A = \frac{1}{2}bh$

$= \frac{1}{2}(12)(9)$

$= 54$

54cm^2

10. A rectangular pool measures 5 yds by 6 yds. A concrete walkway of uniform width is constructed around the pool. The walkway and the pool together cover an area of 72 yd². Set up and solve an equation to show how wide the walkway is.



$A_{walkway} + A_{pool} = 72$

$(2x+5)(2x+6) = 72$

$4x^2 + 12x + 10x + 30 = 72$

$4x^2 + 22x - 42 = 0$

$x = -7$ OR $x = 1.5$

1.5 yds

11. When a driver needs to stop a car, the approximate stopping distance d (in feet) is given by the formula: $d = 0.05v^2 + 2.2v$, where v is the speed of the car (in miles per hour). Suppose a car travels 200 feet before stopping – how fast was the car traveling (to the nearest mph)?

$d = .05v^2 + 2.2v$ $\quad 200 = .05v^2 + 2.2v$

d ↑ stopping dist $\quad$ v ↑ speed

$v = -88.163$ OR $v = 44.963$

≈ 45mph

12. Find three consecutive ODD integers such that the product of the first and third integers minus the middle integer is 338.

$x, x+2, x+4$

$x(x+4) - (x+2) = 338$

$x^2 + 4x - x - 2 = 338$

$x^2 + 3x - 340 = 0$

$x = -20$ OR $x = 17$

17, 19, 21

13. A basketball player shoots the ball with an upward velocity of 20 ft/s, (v_0) The ball is 6 ft. above the floor when it leaves his hand. (h_0) How long will it take for the ball to reach the rim of the basket, 10 ft. above the floor, on its way down?

$h(x) = -16x^2 + 20x + 6$

$10 = -16x^2 + 20x + 6$

(0.25, 10) up $\quad$ (1, 10) ↑ down

1 second

14. Suppose that the equation $V = 25.4x^2 - 512.6x + 4{,}500$ represents the value of a car from 1964 to 2002. What year did the car have the least value? (x = 0 in 1964)

Used calc to find minimum of (10.091, 1913.792)

$x = 0$ in 1964

$10 + 10 = 1974$

1974

15. Which of the following represents the roots to the quadratic equation $b^2 - 16b + 64 = 19$, rounded to the nearest tenth?

$b^2 - 16b + 45 = 0$

used calc to find zeros of (3.64, 0) & (12.36, 0)

$x = 3.6, 12.4$

16. Solve the following system of equations: $\begin{cases} y = 2x + 1 \\ y = 2x^2 - 3 \end{cases}$

$2x^2 - 3 = 2x + 1$

$2x^2 - 2x - 4 = 0$

$2(x^2 - x - 2) = 0$

$2(x + 1)(x - 2) = 0$

$x = -1, x = 2$

$2(-1) + 1 = -2 + 1 = -1$

$2(2) + 1 = 5$

(-1, -1) & (2, 5)

(-1, -1), (2, 5)

17. Each of the "golden arches" at a McDonald's restaurant is in the shape of a parabola. Each arch is modeled by $h(x) = -x^2 + 6x$, where h(x) is the height of the arch (in feet) at a distance x (in feet) from one side.

A. Find the equation of the axis of symmetry. $x = 3$

B. Find the x-intercepts. How long is one arch across its base? (0,0), (6,0) — 6 feet

C. How high is the arch at its highest point? 9 ft

18. Pyro-Tech shot a rocket off going upwards at 60 ft/sec. They shot the rocket from 80 feet above the ground.

V_0 ↑ ft → -16 h_0

A. How long will it take the rocket to reach its highest point? max (1.875, 136.25) — 1.875 seconds

$h(x) = -16x^2 + 60x + 80$

B. How high will the rocket go? 136.25 feet

C. How long after the launch will the rocket return to the ground? 4.793 seconds

zero (4.793, 0)

Bonus: Starting with the standard form of quadratic equation, $ax^2+bx+c=0$, use completing the square to derive the proof of the quadratic formula.