Savings Account

NAME ___________________________

The ability to predict the future is an invaluable task when dealing with money. Who would have guessed gas would quadruple in price in a span of ten years? While making predictions about future gas prices is a nearly impossible task, when investing money in a bank account, the future value can be predicted with a great deal of certainty.

For example, suppose you have $2,000 and hide it under your mattress for 40 years. At the end of 40 years, you would still have $2,000. However, if you had invested it in a bank at an interest rate of 4.5%, you would have more than $12,000 at the end of the 40 years. How is this possible? The answer is *compound interest*, which works in the following way. Money is first invested. Then, at regular intervals (for example, monthly, quarterly, yearly), interest is awarded to the account and becomes the investor's money. In this way, interest is earned on previously earned interest – in other words, the interest is *compounded*.

The table below shows banking amounts for a $100 investment that earns 3% interest each year. Complete the table.

YEARS INVESTMENT HAS BEEN IN THE BANK	BALANCE AT THE START OF THE CURRENT YEAR	INTEREST EARNED FOR THE YEAR (3%)	BALANCE AT THE END OF THE CURRENT YEAR
1	100.00	3.00	103.00
2	103.00	3.09	106.09
3			
4			
5			

The table below is the same as the one above, but uses algebraic notation. Complete this table and work with your neighbor to develop a formula that could be used to find the bank amount for any year.

YEARS INVESTMENT HAS BEEN IN THE BANK	BALANCE AT THE START OF THE CURRENT YEAR	INTEREST EARNED FOR THE YEAR (3%)	BALANCE AT THE END OF THE CURRENT YEAR	
			PREVIOUS BALANCE + INTEREST	SIMPLIFIED AMOUNT
1	P	$0.03P$	$P + .03P$	$P \times (1.03)$
2	$P \times (1.03)$	$0.03[P\times(1.03)]$	$P \times (1.03) + 0.03[P \times (1.03)]$	
3				
4				
5				

The formula for the bank amount for any year is $A = P(1+r)^t$, where t = number of years the investment has been in the bank, P = original amount invested, and r = the interest rate expressed as a decimal.

1. a) Why does the formula use 1 + r?

b) Use this formula to compute the value of the original $2,000 investment after 40 years at an interest rate of 3% if no additional monthly contributions are made.

Now consider, what happens if the bank compounds the interest four times a year, or twice a year. The formula is changed to $A = P\left(1+\frac{r}{n}\right)^{nt}$, where n is the number of times per year the amount is compounded. (Remember: These formulas are for the situation in which no additional money is being contributed by the investor.)

2. In the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$,

a) What does the fraction $\frac{r}{n}$ represent?

b) What does the exponent nt represent?

3. a) Use this new formula to compute the value of the original $2,000 investment after 40 years at an interest rate of 3% if the interest is compounded four times a year and if no additional monthly contributions are made.

b) Notice the answer to Question 3.a) is greater than the amount calculated for Question 1 which invested the same $2,000 for 40 years at 3% interest compounded yearly. Why?

People generally contribute a monthly amount to their savings. This results in the money growing much faster than just waiting for a lump sum to grow to the desired value.

4. **a)** To observe this, use the Compound Interest Simulator to determine how long it will take an investment of $1,000 at 5% to reach $2,000 without any monthly contributions.

b) Now, use the simulator to determine how long it will take the same investment of $1,000 at 5% to reach $2,000 when the investor also makes monthly contributions of $50.

To further observe the effects of compounding interest, imagine we have two people who start saving for retirement.

Person A invests $2,000 at age 30 and then makes a monthly contribution of $200 until age 65; the account has an annual interest rate of 4.5%. Person B executes the same plan, but begins at age 40. This means she only has 25 years of investing compared to person A's 35 years.

While ten years may not seem like much, in terms of compound interest, it is. Using the simulator, determine how much her delay of ten year will have cost person B when she retires.

5. Use the simulator to explore how much money you would need to invest to have $1 million by the time you reach age 65. Try to do this with and without monthly contributions. Try different interest rates as well. Find at least three combinations that yield $1 million, and record them in the table below.

PRINCIPAL ($)	MONTHLY CONTRIBUTION ($)	ANNUAL INTEREST RATE (%)	LENGTH OF INVESTMENT

6. How long will it take an investment of $10,000 to reach $1 million if the rate of return is 10% with no additional contributions?

7. What rate of return would be necessary with an investment of $10,000 to make $1 million by the time you reach age 35?

8. Use the Internet to find the current interest rates for online banks and traditional banks (like the one down the street). Use this information to compare the investment results after 20 years on a $40,000 investment.

BANK NAME	INTEREST RATE	BALANCE OF $40,000 INVESTMENT AFTER 20 YEARS

9. Search online or talk to a banker to find the answer to the following question: What's the difference between annual *rate* and annual *yield*?

10. At what age should you start investing your money?