What is the vertex of the function below?

$$f(x) = 2x^2 - 6x + 8$$

Reminder on axis of symmetry:

Find the vertex of the parabolas given by the equations below.

1. $y = 3x^2 - 24x - 7$

2. $y = x^2 + 6x + 3$

3. $y = -2x^2 - 8x + 10$

4. $y = 2x^2 - 16x + 1$

5. $y = 3x^2 - 24x - 15$

6. $y = -x^2 + 5x + 1$

7. $y = -4x^2 + 8x - 1$

(1, 3)

8. $y = 6x^2 + 12x + 6$

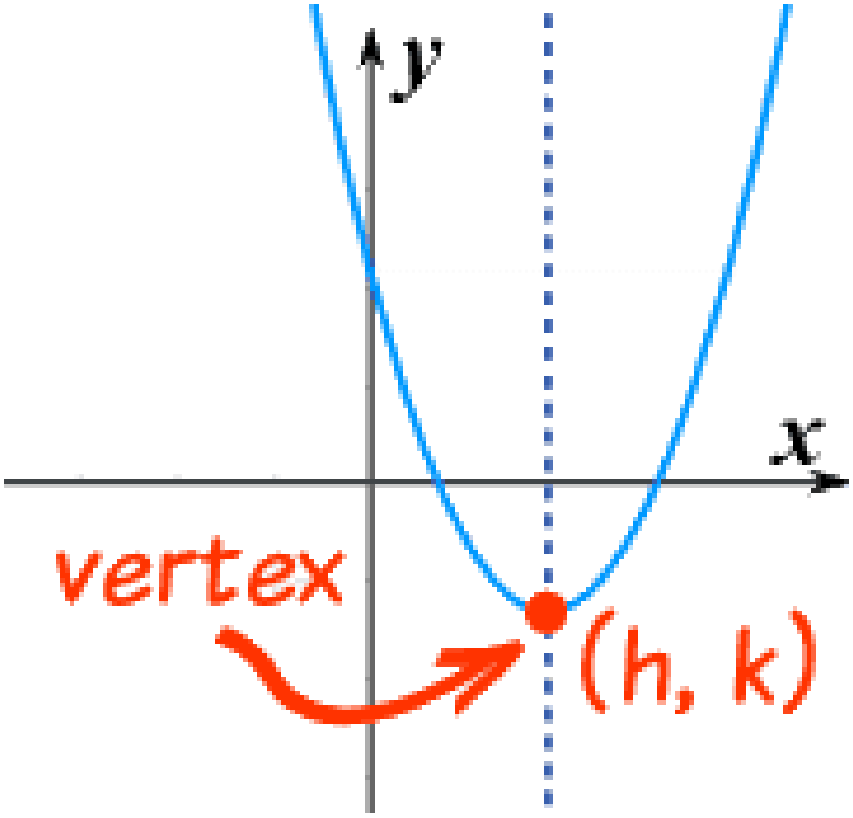
(-1, 0)

9. $y = 2x^2 - 2x + 5$

(0.5, 4.5)

Recall

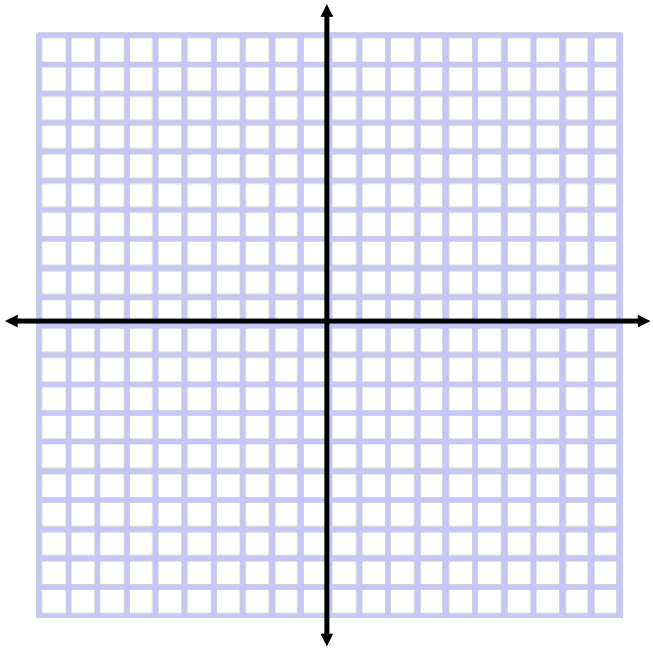
The vertex of a quadratic is identified by the ordered pair (h, k)



where h = $\frac{-b}{2a}$

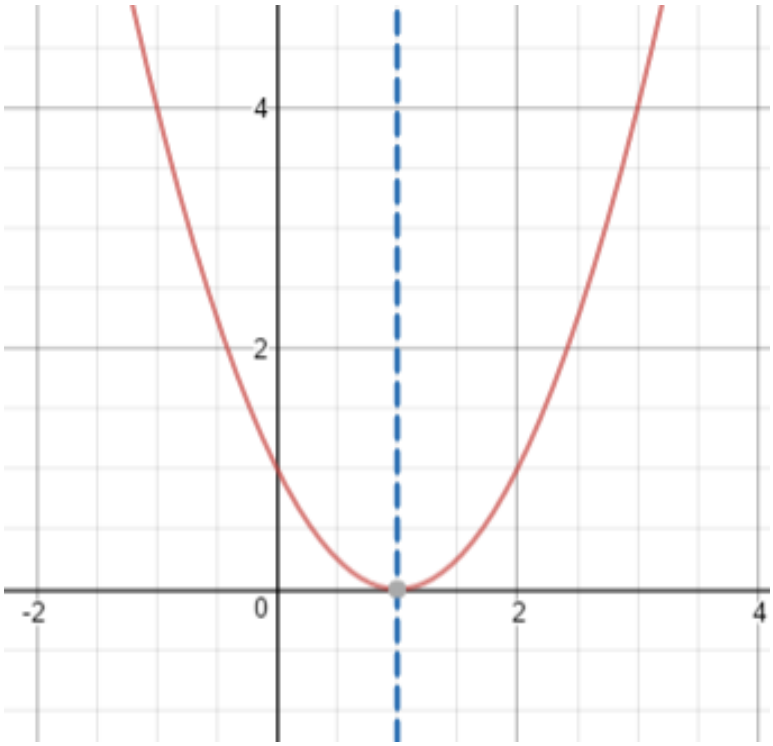
Almost all of the parabolas that we graph will have zeros, but if the parabola does not have a zero, pick an x-value and evaluate it in the function.

$$f(x) = 2x^2 - 6x + 8$$



Axis of Symmetry

x = 1



The axis of symmetry is an axis that shows the symmetry of a parabola.

It is an imaginary vertical line that "cuts the parabola in half," thus it always must pass through the vertex.

Because all parabolas are concave up/down, for parabolas, ALL axes of symmetry are vertical lines.

Thus, they are defined by the equation x = ___.

Axis of Symmetry

$$x = \frac{-b}{2a}$$

Vertex Form

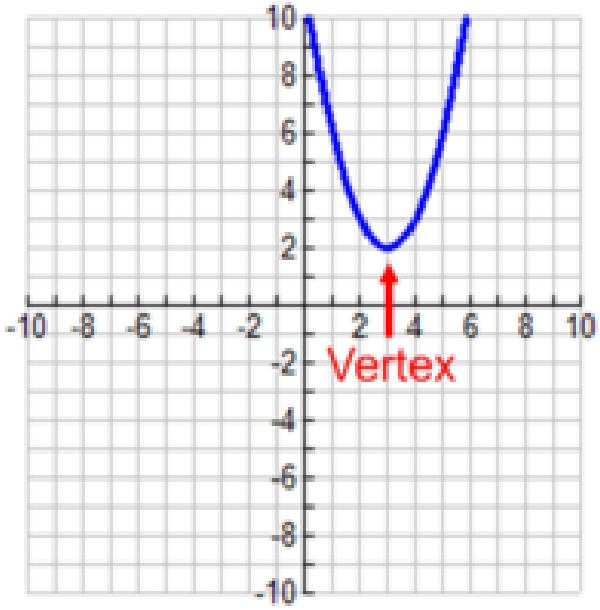
$$y = a(x - h) + k$$

Vertex Form

$y = (x - 3)^2 + 2$

h k

Vertex: (h, k)
Vertex: (3, 2)



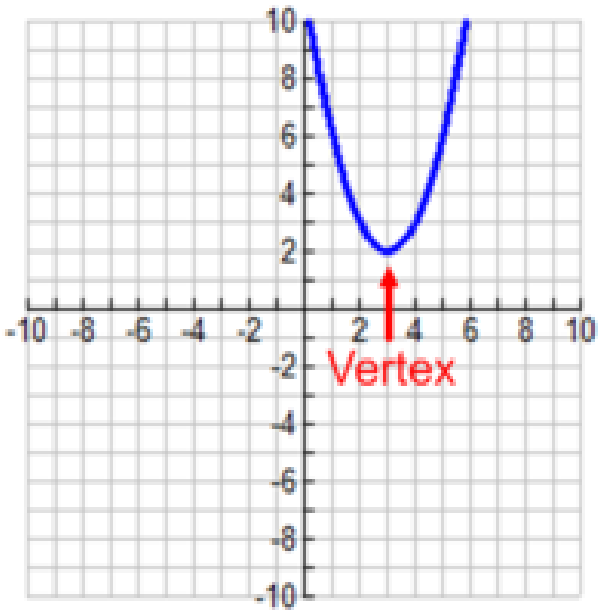
Translating to Standard Form is EASY!!!

Vertex Form

$y = (x - 3)^2 + 2$

h k

Vertex: (h, k)
Vertex: (3, 2)



But how do we go from standard form back to vertex form algebraically!?!?!?

hmmm...

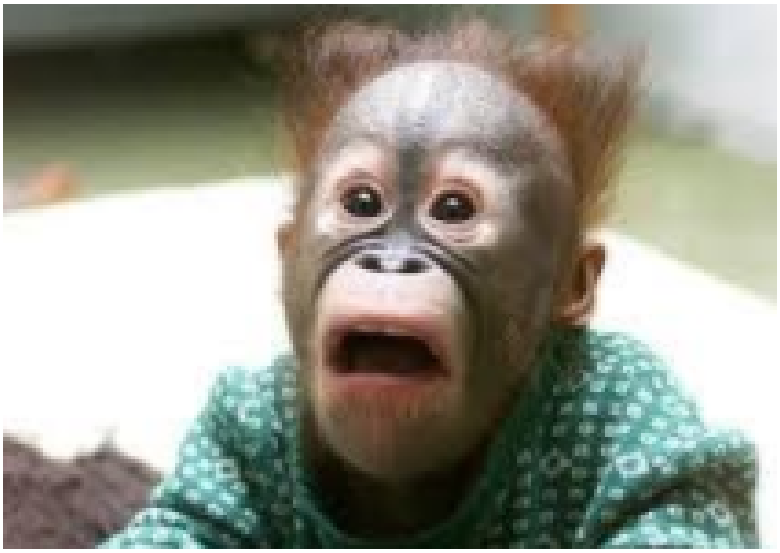
$$y = x^2 - 6x + 11$$

This is done by

COMPLETING THE SQUARE!!!

Before we complete the square, we've got to get our minds thinking of how to create PERFECT SQUARE TRINOMIALS!!!!

So we can create some perfect squares!!!!!!!!! AND THEN COMPLETE THEM!!!!!!!!!



Perfect Square Trinomials

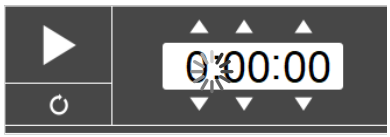
$$4x^2 + 12x + 9$$

$$=$$

$$(2x + 3)^2$$

What value would you need k to be to make the quadratic a perfect square trinomial?

Ex: for $f(x)=x^2-6x+k$ $k=9$ because half of -6 is -3 and $(-3)^2=9$



1.) $f(x)=x^2+8x+k$

2.) $f(x)=x^2-14x+k$

3.) $f(x)=4x^2+20x+k$

To create the perfect square trinomial AKA COMPLETING THE SQUARE,

if you have $x^2 + bx$ you can complete the square by adding

$$\left(\frac{b}{2}\right)^2$$

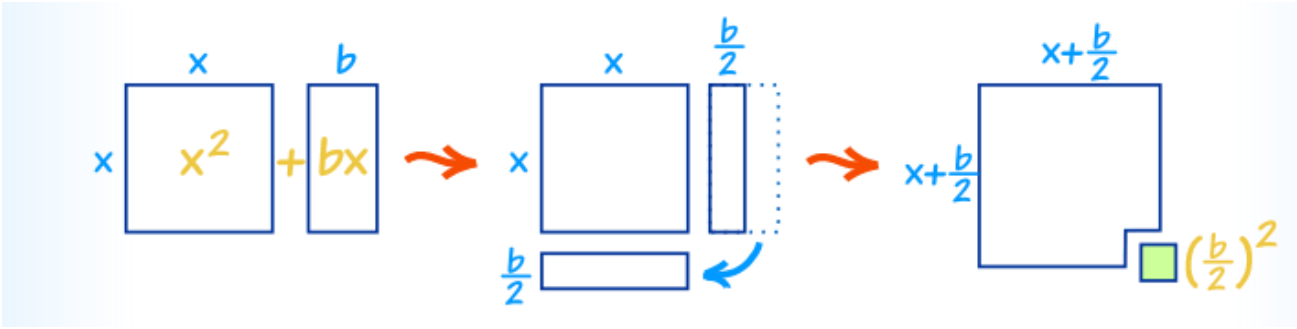
Complete the Square

$+\left(\frac{b}{2}\right)^2$

1.) $f(x)=x^2+8x+k$

2.) $f(x)=x^2-14x+k$

3.) $f(x)=4x^2+20x+k$



As you can see $x^2 + bx$ can be rearranged ***nearly*** into a square ...

... and we can **complete the square** with $(b/2)^2$

In Algebra it looks like this:

$$x^2 + bx \quad + (b/2)^2 \quad = \quad (x+b/2)^2$$

"Complete the Square"

So, by adding $(b/2)^2$ we can complete the square.

And $(x+b/2)^2$ has x only **once**, which is easier to use.

What is the value of c if $x^2 - 24x + c$ is a perfect square?

What is the value of c if $x^2 + 18x + c$ is a perfect square?

Applying Completing the Square

Back to the task at hand!!! How can we change this back into Vertex Form!?

$$y = x^2 - 6x + 11$$

Key Ways to Prevent Careless Errors!

1. ALWAYS, ALWAYS, ALWAYS solve your equation so that you have $x^2 + bx$ ALONE.

2. Notice in the statement above the first term is x^2 NOT ax^2. You MUST GET RID OF a!!!

3. WHATEVER YOU DO TO ONE SIDE OF THE EQUATION, YOU MUST DO TO THE OTHER!

Complete the Square to Change into Vertex Form:

$$f(x) = 3x^2 + 12x - 6$$

$+6 \qquad +6$

$$\frac{f(x)+6}{3} = \frac{3x^2+12x}{3}$$

$$\frac{f(x)+6}{3} = x^2+4x+4$$

$+4$

$$\frac{f(x)+6}{3}+4 = (x+2)^2-4$$

-4

$$\frac{f(x)+6}{3} = 3(x+2)^2-4(3)$$

$$f(x)+6 = 3(x+2)^2-12$$

$-6 \qquad -6$

$$f(x) = 3(x+2)^2-18$$

$(-2,-18)$

You Try!

Complete the Square to Change into Vertex Form:

$$y = g^2 - 2g - 323$$

$$y = w^2 - 6w - 35$$

$$y = 3x^2 - 12x - 7$$